Name: Feedback

SA402 · Dynamic and Stochastic Models　　　　Fall 2022 · Uhan

Exam 1 – 10/17/2022

Instructions

- You have 50 minutes to complete this exam.
- You may use your plebe-issue TI-36X Pro calculator.
- You may not use any other materials.
- **No collaboration allowed.** All work must be your own.
- **Show all your work.** To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.
- Keep this booklet intact.
- **Do not discuss the contents of this exam with any midshipmen until it is returned to you.**

Problem	Weight	Score
1a	1	
1b	1	
1c	1	
1d	1	
2a	1	
2b	1	
3a	1	
3b	1	
4	1	
5	1	
Total		/ 100

Problem 0. Copy and sign the honor statement below. This exam will not be graded without a signed honor statement.

> The Naval Service I am a part of is bound by honor and integrity. I will not compromise our values by giving or receiving unauthorized help on this exam.

Problem 1. You have been hired as an operations research analyst at Poisson Playland, a small amusement park just outside of Simplexville. According to your predecessor's notes, vehicles arrive at the entrance of the parking lot according to a stationary Poisson process at a rate of 17 per hour between 09:00 and 21:00.

a. What is the probability that 100 or fewer vehicles arrive at or before 15:00, given that exactly 75 vehicles arrive between 9:00 and 13:00?

See Lesson 4 for a number of similar examples. Be careful with arithmetic when transforming your probability statement so that you can use independent increments!

b. What is the expected time of the 80th vehicle arrival?

Some of you computed this indirectly, using $E[Y_t] = \lambda t$. Note that you can also compute this directly, using $E[T_n] = n/\lambda$. See pages 2-3 of Lesson 4 for details.

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c. What is the expected number of vehicles by the end of the day (09:00 - 21:00), given that exactly 50 vehicles arrive between 9:00 and 12:00?

See Example 4 in Lesson 4 for a similar example. Be careful with arithmetic when transforming your conditional expected value statement so that you can use stationary increments!

d. Suppose it is 14:00. What is the probability that the next vehicle arrives within 5 minutes (1/12 hour)?

See Problem 1a in the Review Problems for Exam 1 for a similar example.

Problem 2. The ticket booth at Poisson Playland is open from 09:00 to 21:00, and has two queues: one for members only, and one for the general public. Again, according to your predecessor, customers arrive at the members only queue at a rate of 7 per hour, and the general public queue at a rate of 23 per hour. Model the customer arrivals at both queues as independent, stationary Poisson processes.

a. What is the probability that the 49th customer (either member or general public) arrives at the ticket booth at or before 11:00?

   See Example 4 in Lesson 5 for a similar example.

b. What is the probability that 10 or more customers arrive at the members only queue during each of 2 consecutive hours? (For example, 10 or more customers arrive between 9:00 and 10:00, and then 10 or more customers arrive between 10:00 and 11:00.)

   See Problem 1e in the Review Problems for Exam 1 for a similar example.

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Problem 3. After working at Poisson Playland for a few days, you start to suspect that your predecessor was wrong about the parking lot, so you collect your own data. You find that the vehicles actually arrive at the entrance of the parking lot according to a nonstationary Poisson process with the following integrated rate function:

$$\Lambda(\tau) = \begin{cases} 20\tau & \text{if } 0 \leq \tau < 3 \\ 5\tau + 45 & \text{if } 3 \leq \tau < 9 \\ 10\tau & \text{if } 9 \leq \tau \leq 12 \end{cases}$$

where $\tau = 0$ corresponds to 9:00 and $\tau = 12$ corresponds to 21:00.

a. What is the expected number of arrivals between 11:00 and 14:00?

See Examples 1b and 2c in Lesson 6 for similar examples.

b. What is the probability that 23 or more vehicles arrive between 17:00 and 19:00?

See Example 2d in Lesson 6 for a similar but slightly more complex example.

Problem 4. After the parking lot debacle, you start to suspect that your predecessor was wrong about the ticket booth as well. After collecting your own data, you find that customers (members and the general public together) actually arrive at the ticket booth according to a nonstationary Poisson process with the arrival rate function below:

$$\lambda(\tau) = \begin{cases} 32 & \text{if } 0 \le \tau < 3 \\ 8 & \text{if } 3 \le \tau < 9 \\ 12 & \text{if } 9 \le \tau \le 12 \end{cases}$$

where $\tau = 0$ corresponds to 9:00 and $\tau = 12$ corresponds to 21:00.

What is the integrated rate function for this nonstationary Poisson process?

See Example 2a in Lesson 6 for a similar example. Be careful with the limits of integration and the associated integrands when you compute the integrated rate function.

Problem 5. The Simplexville Electric Company is conducting a study of its power line along the busiest part of Main Street. Looking at its historical data, the company has observed that power surges occur at a rate of 12 per hour, and this rate does not change over time. However, it also has noticed that the power surges come in "waves": a large power surge is always followed by a smaller power surge exactly 1 minute later.

Professor I. M. Wright is consulting for the electric company. He thinks that the occurrence of power surges (both large and small) satisfies the independent increments property. Is he correct? Briefly explain.

Hint. It turns out that Professor I. M. Wright was your predecessor at Poisson Playland!

Note that the problem only asks about the independent increments property. See page 6 of Lesson 4 for details on how the independent increments property manifests in an arrival counting process.